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BURSTINESS DESCRIPTORS OF TRAFFIC STREAMS: INDICES OF DISPERSION AND PEAKEDNESS

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Abstract

The burstiness of a traffic stream can have a profound effect on performance statistics in the queue to which the stream is offered, such as line length, waiting times and blocking probabilities. Mathematical descriptors quantifying burstiness should be versatile enough to characterize burstiness, yet analytically tractable to permit numerical calculation. This paper surveys two important descriptors of burstiness: indices of dispersion and peakedness. It discusses their effect on queueing statistics and illustrates them via an example from the domain of TES traffic streams. A conclusion of this example is that peakedness is more general than indices of dispersion.

1 Introduction

The effect of burstiness on queueing systems has been long recognized by telephone traffic engineers and other workers [4, 14, 15]. Intuitively, burstiness is present in a discrete traffic process, if its arrival points appear to form visual clusters. The mathematical underpinning of burstiness are less obvious. If $\{X_n\}$ is the time series of interarrival times, then two main sources of burstiness are due to the shapes of the marginal distribution and autocorrelation function of $\{X_n\}$. For example, burstiness tends to increase with the coefficient of variation of the marginal distribution of $\{X_n\}$, and also in the presence of strong positive autocorrelations therein. There seems to be, however, no single widely-accepted notion of burstiness in the literature.

This paper surveys two important descriptors of burstiness: indices of dispersion and peakedness; these can also be viewed as broad characterizations of general traffic processes. Their advantages lie in the relative ease of obtaining the data required for their calculation, and in the facility of the corresponding mathematical analysis. Most importantly, these descriptors appear to capture the effect of the intuitive notion of burstiness on important performance statistics of queues, such as line length, waiting times and blocking probabilities in finite-buffer queues.

To illustrate the functional form, we exhibit these descriptors for the class of TES$^+$ processes — a versatile class of stochastic sequences, designed to capture, simultaneously, the marginal distribution and autocorrelation function of empirical data. This example shows that the corresponding busrtiness descriptors are given in terms of first-order and second-order properties of the interarrival-time process.

2 The PTC and ISE Traffic Principles

The two burstiness descriptors, indices of dispersion and peakedness, are motivated, respectively, by two traffic principles; the Poisson Traffic Comparison (PTC) principle, and the Infinite-Server Effect (ISE) principle. These principles trace their origin to teletraffic analysis and design, and will be explained next.

The PTC principle compares a traffic-related descriptor (statistic), d_X, of a target traffic process, $\{X_n\}$, to the corresponding descriptor, d_E, of a "comparable" Poisson process, $\{E_n\}$, where the E_n are iid exponential random variables. The descriptor statistic is often related to moments of the underlying process. The PTC principle adopts the view that Poisson processes constitute a class of traffic benchmarks; indeed, these classic traffic processes have a long history in teletraffic theory and practice. Furthermore, "comparability" usually means that the benchmark Poisson process is chosen to have the same rate as the target traffic process. The mathematical comparison is most often performed by taking the target-to-benchmark descriptor ratio. In particular, under the PTC principle, a generic burstiness descriptor, b_X, has the form

$$b_X = \frac{d_X}{d_E}. \quad (2.1)$$

Eq. (2.1) has the additional attraction that PTC-based descriptors of any Poisson process trivially evaluate to unity, as befits a benchmark. The indices of dispersion descriptors of burstiness, treated in Section 3, are motivated by the PTC principle.

The ISE principle adopts the view that to gauge an aspect of a target traffic stream, $\{X_n\}$, one offers that stream to an infinite-server system and observes the stream effect via a suitable statistic, d_X; infinite-server systems are chosen due to their relative tractability. For example, to characterize burstiness, a suitable description would be the equilibrium moments of waiting times or number of customers in the system. The PTC principle is often employed implicitly, by using a normalized form of the description in such a way that it evaluates to unity for any Poisson process. The ISE principle is also used to construct an approximation to a target traffic stream via a mathematically simpler stream. This is attained by equating equilibrium moments of the number of busy servers induced by the two streams. An example is the "three moment match" for creating an interrupted Poisson stream approximation to overflow traffic [13]. The burstiness notion of peakedness, treated in Section 4, is subject to the ISE principle.

3 Indices of Dispersion

Consider a stationary, non-negative random sequence, $\{X_n\}_{n=0}^{\infty}$, interpreted as the interarrival times of a traffic process. Let the common distribution function of the X_n be denoted by $F_X(x)$; similarly, denote $\lambda_X = 1/E[X_n]$ (traffic rate), $\sigma_X^2 = Var[X_n]$, and $c_X = \lambda_X \sigma_X$. We assume that $0 < \sigma_X < \infty$, and that $\{X_n\}$ is simple, namely, $P\{X_n = 0\} = 0$. The coefficient of variation, c_X, can be used to characterize traffic burstiness, albeit rather weakly, since it utilizes just first-order statistics of $\{X_n\}$ (those pertaining to $F_X(x)$ only). In contrast, the second-order properties are characterized by the autocorrelation function of $\{X_n\}$, given by

$$\rho_X(j) = \frac{E[X_k\, X_{k+j}] - \lambda_X^{-2}}{\sigma_X^2}, \quad n = 0, 1, \ldots, \quad (3.1)$$

which measures temporal linear dependence among lagged interarrival times, and will prove to be a key ingredient in the burstiness descriptors to be discussed in the sequel.

The index of dispersion for intervals (IDI) [7], associated with $\{X_n\}$ is a burstiness descriptor defined by

$$J_X(n) = \frac{Var[\sum_{j=1}^{n} X_j]}{n\, \lambda_X^{-2}}. \quad (3.2)$$

In order to relate the IDI to the PTC principle, denote $v_X(n) = Var[\sum_{j=1}^{n} X_j]$, and note that for a Poisson process with interarrival times, $\{E_n\}$, of mean $1/\lambda_X$, one has $v_E(n) = n\, \lambda_X^{-2}$. Thus, Eq. (3.2) is the ratio, $J_X(n) = v_X(n)/v_E(n)$, in adherence to the PTC principle. A simple computation provides the explicit relation of $J_X(n)$ to c_X and ρ_X, namely,

$$J_X(n) = c_X^2 \left[1 + 2 \sum_{j=1}^{n-1} \left(1 - \frac{j}{n} \right) \rho_X(j) \right]. \quad (3.3)$$

Let $\{N(t)\}$ be the equilibrium counting process of arrivals in the interval $(0, t]$, so that $E[N(t)] = \lambda_X t$. A related burstiness descriptor is the index of dispersion for counts (IDC) [7], associated with $\{X_n\}$, and given by

$$I_X(t) = \frac{Var[N(t)]}{E[N(t)]} = \frac{Var[N(t)]}{\lambda_X t} \quad t \geq 0, \quad (3.4)$$

That the IDC adheres to the PTC principle follows from the fact that the mean and variance of a Poisson counting process are equal.

Of particular interest are the limiting indices of dispersion, $J_X = \lim_{n\to\infty} J_X(n)$ and $I_X = \lim_{t\to\infty} I_X(t)$. It can be shown that [7]

$$J_X = I_X = c_X^2 \left[1 + 2 \sum_{j=1}^{\infty} \rho_X(j) \right]. \quad (3.5)$$

The indices of dipersion, (3.2)–(3.5), are more satisfactory descriptors of traffic variability (burstiness) than the coefficient of variation, c_X, since they take into account the autocorrelation function, $\rho_X(j)$, as well. In fact, for renewal traffic, $J_X(n) \equiv J_X = I_X = c_X^2$. Moreover, these indices of dispersion may be usefully applied to the study of the effect of an offered load on a server system [16, 4, 7].

In practical queueing studies, one uses the indices of dispersion, $J_X(n)$ and $I_X(t)$, and especially the simpler common limit $J_X = I_X$, by estimating their values from empirical observations and then fitting a suitable traffic model, such as a Markov modulated Poisson process (MMPP) which is consistent with the observed index of dispersion [7]. Having thus constructed a model, a queueing system is analyzed with this traffic as the offered load. While this approach has led to some useful results, the index-of-dispersion characterization of traffic burstiness is still fairly weak from a queueing viewpoint. The reason is that indices of dispersion do not consider server systems in their definition. This situation will be remedied in the next section which will introduce a more powerful traffic burstiness characterization.

4 The Peakedness Functional

Peakedness is a traffic burstiness descriptor, based on the ISE principle. It gauges traffic "smoothness" via its effect on an iid infinite-server group with common service distribution $B(x)$ (usually the exponential distribution) and mean $1/\mu_B$ (μ_B is the service rate.) The statistic of interest is the equilibrium expectation of $S(t)$ — the number of busy servers at time t. R. Wilkinson [17] had introduced the peakedness concept in teletraffic theory in order to characterize the action of superposition of overflow streams when offered to a secondary trunk group. The object was to compute the grade of service (blocking probability) in the context of his "equivalent random method" [8]. A simple and more flexible computational method utilizing peakedness for grade-of-service computatation was introduced later by W.S. Hayward (*ibid.*) An advantage of peakedness is that it may be used to approximate various statistics in a single-server system, with or without finite buffers, such as waiting times and blocking probabilities [3, 8].

Let $a = \lambda_X/\mu_B$, and note that Little's Law implies $\lim_{t\uparrow\infty} E[S(t)] = a$. The peakedness functional, $z_X[B]$, maps the service time distribution, $B(x)$, to the following scalar [17]

$$z_X[B] \;=\; \lim_{t\uparrow\infty} \frac{Var[S(t)]}{E[S(t)]} \;=\; \frac{\sigma_S^2}{a}. \qquad (4.1)$$

For any service distribution, $B(x)$, define $B_0(x) = B(x/\mu_B)$, so that $B_0(x)$ has unit service rate. Define further $B_\mu(x) = B_0(\mu x)$ to be a family of service time distribution functions, parameterized by $\mu > 0$. The peakedness function associated with this parametric family is

$$z_{X,B}(\mu) \;=\; z_X[B_\mu]. \qquad (4.2)$$

An important formula for $z_{X,B}(\mu)$ was obtained by A.E. Eckberg in [3]. Let $M_X(t)$ be the expectation function of the traffic process, $\{X_n\}$, that is, the mean number of arrivals in $(0, t]$ when the origin is an arrival point; thus, $M_X(t)$ is the analog of the renewal function in renewal theory. Let $B^c(x)$ designate the complementary service time distribution, that is, $B^c(x) = 1 - B(x)$, and let $\psi_B(x)$ denote the correlation function

$$\psi_B(t) \;=\; \int_0^\infty B^c(u)\, B^c(t+u)\, du, \qquad (4.3)$$

of $B^c(x)$. Then one has [3]

$$z_{X,B}(\mu) \;=\; 1 - \frac{\lambda_X}{\mu} + 2\mu \int_0^\infty \psi_B(t)\, dM_X(t). \quad (4.4)$$

Furthermore, if the density of the expectation function, $m_X(t) = dM_X(t)/dt$, exists then one may also write

$$z_{X,B}(\mu) \;=\; 1 - \frac{\lambda_X}{\mu} + 2\mu \int_0^\infty \psi_B(t)\, m_X(t)\, dt. \quad (4.5)$$

An important special case occurs for exponential service time distributions, $B(x) = 1 - e^{-\mu x}$. The notation $z_{X,exp}(\mu)$ and $\psi_{exp}(x)$ will be used for this case. Since

$$\psi_{exp}(y) \;=\; \frac{1}{2\,\mu}\, e^{-\mu y}, \qquad (4.6)$$

it follows that

$$z_{X,exp}(\mu) \;=\; 1 - \frac{\lambda_X}{\mu} + \int_0^\infty e^{-\mu t}\, m_X(t)\, dt. \quad (4.7)$$

Letting $\tilde{f}(s) = \int_0^\infty e^{-sx} f(x)\, dx$ denote the Laplace transform of a function, $f(x)$, one has

$$z_{X,exp}(\mu) \;=\; 1 - \frac{\lambda_X}{\mu} + \tilde{m}_X(\mu)\, . \qquad (4.8)$$

Eq. (4.8) has an important consequence. It implies that when $\tilde{m}(s)$ does not depend on μ, then knowledge of $z_{X,exp}(\mu)$ suffices to determine $z_{X,B}(\mu)$ for *any* service distribution $B(x)$. To see that, observe that from $z_{X,exp}(\mu)$, one can determine $\tilde{m}_X(\mu)$, from which $m_X(t)$ is obtained by inversion. Eq. (4.5) can now be used to obtain $z_{X,B}(\mu)$ with the aid of $m_X(t)$.

Practical applications of the peakedness function $z_B(\mu)$ to queueing problems are found in [8, 3, 5].

5 The Peakedness and IDI of TES$^+$ Traffic Streams

This section illustrates the peakedness function and IDI for a subclass of a new versatile class of random sequences, called TES (*Transform-Expand-Sample*); for details see [9, 10]. It also will show that the former contains more information than the latter.

In essence, TES is a modulo-1 reduction of a simple linear autoregressive scheme, followed by additional transformations. Furthermore, while the basic TES formulation is Markovian, these transformations usually result in non-Markovian processes. The TES modeling methodology is a non-parametric approach which fits a model to empirical records, whose underlying probability law is assumed to be stationary, by simultaneously capturing both first-order (histogram) and second-order (autocorrelation) empirical statistics. TES enjoys a number of key advantages: it can fit arbitrary marginal distributions; it can generate a wide variety of sample path behaviors, which in turn give rise to autocorrelation functions with diverse functional forms; TES sequences are easily generated on a computer, and their autocorrelation functions can be accurately computed numerically at a modest computational cost. The principal modeling activity consists of a heuristic search for a suitable dependence structure under software support;

see [6] for a description of the visual interactive software environment, TEStool.

Let $\{V_n\}_{n=1}^{\infty}$ be a sequence of iid (independent identically distributed) random variables, called the *innovation sequence*, and let U_0 be uniform on $[0,1)$ and independent of the innovation sequence. A *background* TES$^+$ process is defined by a recursive scheme of the form

$$U_n^+ \;=\; \begin{cases} U_0, & n = 0 \\ \langle U_{n-1}^+ \;+\; V_n \rangle, & n > 0 \end{cases} \tag{5.1}$$

in which the angular brackets indicate the fractional part operator, namely, $\langle x \rangle = x - \max\{\text{integer } n : \; n \le x\}$. The superscript notation in (5.1) is motivated by the fact that $\{U_n^+\}$ can generate lag-1 autocorrelations in the range [0,1]; a corresponding class, TES$^-$, with negative lag-1 autocorrelations can also be defined [9], but will not be treated here. $\{U_n^+\}$ can be shown to be a Markovian sequence with uniform $[0,1)$ marginals, *regardless* of the innovations selected; however, the common marginal density, f_V, of the innovation sequence determines the corresponding temporal dependence, and in particular, the attendant autocorrelation function.

A Lebesgue-measurable function, $D(x)$, from $[0,1)$ to the reals is called a *distortion*. In order to compute its Laplace transform, $\tilde{D}(s)$, one extends this definition to $(1,\infty)$ by setting $D(x) = 0$ for $x > 1$. A transformed random sequence, $\{X_n^+\}_{n=0}^{\infty}$, of the form

$$X_n^+ \;=\; D(U_n^+), \tag{5.2}$$

is called a *foreground* TES$^+$ process.

Let $\tilde{f}_V(s)$ denote the Laplace transform of $f_V(x)$, and let $g_U^+(v|u)$ be the 1-step transition density of $\{U_n^+\}$. Then g_U^+ has the representation [9]

$$g_U^+(v|u) \;=\; \sum_{\nu=-\infty}^{\infty} \tilde{f}_V(i2\pi\nu)\, e^{i2\pi\nu(v-u)} \tag{5.3}$$

Furthermore, the autocorrelation function, $\rho_X^+(\tau)$, of $\{X_n^+\}$ has the representation [9]

$$\rho_X^+(\tau) \;=\; \frac{1}{(\sigma_X^+)^2} \sum_{\substack{\nu=-\infty \\ \nu\neq 0}}^{\infty} \tilde{f}_V^{\tau}(i2\pi\nu)\, |\tilde{D}(i2\pi\nu)|^2, \quad \tau \ge 0. \tag{5.4}$$

In general, $\{X_n^+\}$ may or may not be Markovian, depending on $D(x)$, but in this paper, we confine ourselves to monotone distortions of the form

$$D(x) \;=\; F_X^{-1}(x), \tag{5.5}$$

where $F_X(x)$ is some prescribed distribution function. The inversion method [1] then ensures that the random variables, $X_n^+ = F_X^{-1}(U_n^+)$, have the common marginal distribution $F_X(x)$.

We now proceed to calculate the peakedness function of TES$^+$ traffic streams, namely, those with an interarrival interval sequence, $\{X_n\} = \{X_n^+\}$, with a distortion of the form (5.5); since, $D = F_X^{-1}$ is monotone, in this case $\{X_n^+\}$ is also Markovian. Let $f_1^+(y|x)$ be the 1-step transition density of X_n^+ given $X_{n-1}^+ = x$, and let $m_X^+(t|x)$ denote the corresponding conditional expectation density given the initial state $X_0^+ = x$, with Laplace transform $\tilde{m}_X^+(s|x)$. The traffic rate is denoted by λ_X^+ and similarly for other quantities associated with $\{X_n^+\}$.

The auxiliary peakedness function, $z_{exp}^+(\mu, x)$, is defined by

$$z_{exp}^+(\mu, x) \;=\; 1 \;-\; \frac{\lambda_X}{\mu} \;+\; \tilde{m}_X^+(\mu|x). \tag{5.6}$$

The auxiliary peakedness function, $z_{exp}^+(\mu, x)$, is related to the peakedness functional $z_{exp}^+(\mu)$ by the simple relation

$$z_{exp}^+(\mu) \;=\; \int_0^1 z_{exp}^+(\mu, D(u))\, du, \tag{5.7}$$

since conditioning on $X_0^+ = D(u)$ is equivalent to conditioning on $U_0^+ = u$, the latter being uniform on $[0,1)$. It can be shown [11] that $z_{exp}^+(\mu, D(u))$ satisfies the following integral equation

$$z_{exp}^+(\mu, D(u)) \;=\; 1 \;-\; \frac{\lambda_X^+}{\mu} \;+\; \frac{\lambda_X^+}{\mu}\, \tilde{f}_1^+(\mu|D(u)) \;+\; \int_0^1 z_{exp}^+(\mu, D(v))\, e^{-\mu D(v)}\, g_U^+(v|u)\, dv. \tag{5.8}$$

An investigation of the integral equation (5.8) yields the instructive formula

$$z_{exp}^+(0) \;=\; \frac{1 + (c_X^+)^2}{2} \;+\; (c_X^+)^2 \sum_{\tau=1}^{\infty} \rho_X^+(\tau). \tag{5.9}$$

For details, consult [11]. It is known [12] that for a renewal interarrival interval sequence, $\{X_n\}$,

$$z_{X,exp}(0) \;=\; \frac{1 + c_X^2}{2}. \tag{5.10}$$

Therefore, Eq. (5.9) can be viewed as a generalization of the peakedness function at zero, from the renewal case to the (Markovian) TES$^+$ case. In fact, if we select the peakedness value, $z_{X,exp}(0)$, as a descriptor of burstiness, then Eq. (5.9) has an instructive interpretation, as follows. By Eq. (5.10), the first term on the righthand side of (5.9) can be viewed as the burstiness due to the first-order (marginal-related) statistics of $\{X_n^+\}$. In a similar vein, the second term on the righthand side of (5.9) can be viewed as the burstiness due to the second-order (autocorrelation-related) statistics of $\{X_n^+\}$.

Several additional conclusions may be drawn from Eq. (5.9). Observe that positive autocorrelations tend to exacerbate burstiness, while mixed-sign autocorrelations have a mitigating effect on the same. Furthermore, significant autocorrelations will dominate the burstiness of the traffic. This last observation should be taken as a cautionary tale, in that it highlights the modeling risk inherent in modeling only traffic distributions, while ignoring significant autocorrelations therein.

For TES$^+$ traffic processes, there is a simple relation between the peakedness value $z^+_{exp}(0)$ and the corresponding IDI, I^+_X. A comparison of (5.9) with (3.5) reveals that

$$I^+_X \;=\; 2\,z^+_{exp}(0) \;-\; 1. \tag{5.11}$$

Thus the IDI provides the same information as the single peakedness value $z^+_{exp}(0)$. Thus, the peakedness function, $z^+_{exp}(\mu)$, is a more informative characterization of traffic than the index of dispersion concept. In particular, for general traffic $\{X_n\}$, the values of $z_{X,exp}(\mu)$, $\mu > 0$, carry additional information which determines the effect of the traffic on a server system; see [8, 3, 5] for details. To this end, one utilizes the following approximate solution of the integral equation (5.8) for TES$^+$ processes, $\{X^+_n\}$,

$$z^+_{exp}(\mu) \simeq \frac{1}{1-\tilde{f}^+_X(\mu)} - \frac{\lambda^+_X}{\mu} + \frac{\lambda^+_X}{\mu[1-\tilde{f}^+_X(\mu)]} \sum_{\substack{\nu=-\infty \\ \nu\neq 0}}^{\infty} \frac{\tilde{f}_V(i2\pi\nu)}{1-\tilde{f}_V(i2\pi\nu)}\,|d_\nu(\mu)|^2, \tag{5.12}$$

in which $\tilde{f}^+_X$ is the Laplace transform of the marginal density of $\{X^+_n\}$, and $d_\nu(\mu) = \int_0^1 e^{-i2\pi\nu v - \mu D(v)}\,dv$.

We remark that accurate numerical evaluations of $z^+_{exp}(\mu, D(u))$, and subsequently of $z^+_{exp}(\mu)$, utilizing the relation (5.7), can be obtained from the integral equation (5.8) by use of finite rank kernel approximations. The technique is illustrated in [11].

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